

HYDROMAGNETICS OF ADVECTIVE ACCRETION FLOWS AROUND BLACK HOLES: REMOVAL OF ANGULAR MOMENTUM BY LARGE SCALE MAGNETIC STRESSES

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ABSTRACT

We show that the removal of angular momentum is possible in the presence of large scale magnetic stresses in geometrically thick, advective, sub-Keplerian accretion flows around black holes in steady-state, in the complete absence of α -viscosity. The efficiency of such an angular momentum transfer could be equivalent to that of α -viscosity with $\alpha = 0.01 - 0.08$. Nevertheless, required field is well below its equipartition value, leading to a magnetically stable disk flow. This is essentially important in order to describe the hard spectral state of the sources, when the flow is non/sub-Keplerian. We show in our simpler 1.5-dimensional, vertically averaged disk model that larger the vertical-gradient of azimuthal component of magnetic field, stronger the rate of angular momentum transfer is, which in turn may lead to a faster rate of outflowing matter. Finding efficient angular momentum transfer, in black hole disks, via magnetic stresses alone is very interesting, when the generic origin of α -viscosity is still being explored.

Subject headings: accretion, accretion disks — MHD — jets and outflows — X-rays: binaries — galaxies: active

1. INTRODUCTION

Blandford & Payne (1982) showed the possibility of energy and angular momentum removal from a Keplerian accretion disk by the magnetic field lines that extend from the disk surface to large distance. In the framework of infinite conductivity and self-similar model, they showed that the magnetic stresses can extract the angular momentum from a geometrically thin accretion disk, which helps matter to accrete, independent of the presence of viscosity. Furthermore, they argued that such a mechanism is responsible for the observed jets/outflows from accreting sources, when magnetic stresses convert a centrifugal outflow into a collimated jet. The disk matter has been argued to be outflowing through the outgoing field lines. The time evolution of axisymmetric, weak magnetic fields threading geometrically thin, Keplerian accretion disks with finite conductivity in a specific model framework was furthermore investigated by Lubow et al. (1994), however without considering possible angular momentum transfer by the magnetic field. On the other hand, in the presence of infinite conductivity, the magnetic field, in the same model framework which does not consider the contributions from the magnetic stresses, would be kept on amplifying by the accretion of gas, till it stops the accretion (also see Spruit 2013). However, Bisnovatyi-Kogan & Lovelace (2007) and Lovelace et al. (2009) showed that the radially inward flow is possible for plasma- $\beta > 1$ and Prandtl number ≥ 1 , in the stationary channel-type flows having small optical depth in the absence of turbulent viscosity, which also could exhibit electromagnetic outflows for smaller Prandtl number. They showed that the large-scale field keeps drifting inward until a stationary state arises, when the magnetic, centrifugal, and gravitational forces become comparable. This furthermore reveals the flow velocity profile differing significantly from the Keplerian profile.

The idea of exploring magnetic stress in order to explain astrophysical systems was, in fact, implemented much earlier. For example, the solar wind was understood to have decreased Sun's angular momentum through the effect of magnetic stresses (see, e.g., Weber & Davis 1967), the proto-stellar gas clouds might have been contracted by magnetic effects (Mouschovias & Paleologou 1980). In the context of accretion disk, Ozernoy & Usov (1973) and Blandford (1976) showed that the energy is possible to be extracted continuously by electromagnetic torques and twisted field lines. Furthermore, Cao & Spruit (2002) showed, by a linear stability analysis of the accretion disks, that angular momentum is possible to be removed by the magnetic torque exerted by a centrifugally driven wind. The same authors (Cao & Spruit 2013) also discussed that moderately weak fields can cause sufficient angular momentum loss, via a magnetic wind to balance outward diffusion in geometrically thin accretion disks. However, plasma- β has to be much smaller than unity to explain the tendency of strong flux bundles at the centers of disk to stay confined, as seen in numerical simulations. Nevertheless, Ogilvie & Livio (1998) showed, by solving the local vertical structure of a thin accretion disk threaded by a poloidal magnetic field, the shortcoming of launching an outflow and suggested for an existence of additional source of energy for successful launching of the outflow.

However, observationally outflows/jets are mostly found to be emanating from the disk when it is in a hard state (e.g. Belloni et al. 2000), which is non/sub-Keplerian. Note that jets appear to be highly heterogeneous with velocities ranging from a few tens of million cm/sec to the escape velocity from the disk. Superluminal sources, however, appear to exhibit jet velocity around the speed of light (Miley 1980). The jets are found in disks around stellar mass black hole sources (e.g. GRS 1915+105) as well as supermassive black hole sources (e.g. M87).

Therefore, most of the modern models describing outflows/jets from the accretion disks are based on sub-Keplerian,

advective model when the flow has a significant radial velocity, unlike the Keplerian disk. For example, a class of self-similar, advection dominated solutions was proposed by Narayan & Yi (1995), in order to describe bipolar outflows from black hole sources, e.g. Sgr A*. Later on, such a class of advection dominated solution exhibiting outflows/jets was applied in many other contexts, e.g. core-collapsing disks and gamma-ray bursts (Di Matteo et al. 2002), low-radiative-efficiency nuclei of elliptical galaxies (Di Matteo et al. 2002).

In a different model framework, Chakrabarti & Titarchuk (1995), Chakrabarti (1999), Chakrabarti & Manickam (2000) described advective, sub-Keplerian accretion flows in order to explain outflows, quasi-periodic oscillations (QPOs) and spectral states in black hole sources. Furthermore, Mukhopadhyay (2003), Mukhopadhyay & Ghosh (2003), Rajesh & Mukhopadhyay (2010a) described general advective accretion flows (GAAFs) around black holes and neutron stars and showed the effects of rotation of the black hole on to the solutions. The last authors also included various cooling effects explicitly and showed how the solutions get affected by the cooling properties.

However, all the above models were formulated in the framework of Shakura-Sunyaev α -viscosity (Shakura & Sunyaev 1973), when the flows are assumed to be embedded with the plasma- $\beta \gg 1$. Hence, the matter transport is assumed to be supported via turbulent viscosity, not by large scale magnetic field, unlike that chosen by Blandford & Payne (1982). Nevertheless, Bhattacharya et al. (2010) showed that the transport is also possible in the presence of outflow in a 2.5-dimensional accreting system; it does not matter whether the outflow is magnetic or hydrodynamic. Note that outflows and even jets can also be formed in the absence of magnetic field. This is likely to occur when the flow is radiation trapped and the accretion rate is super-Eddington or super-critical (Lovelace et al. 1994; Begelman et al. 2006; Febrika 2004; Ghosh & Mukhopadhyay 2009).

The 2.5-dimensional accretion model, proposed by Bhattacharya et al. (2010), will be complete if the effects of (large scale) magnetic field is included therein. In that case, one presumably can explain outflow of matter plunging through the outgoing magnetic field lines more spontaneously, as Blandford & Payne (1982) did in the Keplerian framework. To the best of our knowledge, so far there is no attempt to obtain a self-consistent set of advective disk-outflow coupled accretion solutions in the presence of large scale magnetic field, which has lots of implication to explain low/hard state of sources. This, however, has been discussed in some extent for circumstellar disks around young stars (see, e.g., Königl & Salmeron 2011), without discussing the detailed solutions of all the dynamical variables. The present work steps forward in order to obtain such a set of solutions for black holes.

In various numerical set ups, magnetohydrodynamic (MHD) simulations of accretion on to magnetized compact objects have already been explored. As examples, some of them considered axisymmetric systems in the presence of magnetosphere (Romanova et al. 2011), some other aimed at investigating advection of matter and magnetic field in the turbulent/diffusive disks (Dyda et al. 2013), when the field strength decreases due to reconnection and annihilation at a later time. Other groups, explored general relativistic magnetohydrodynamic (GRMHD) simulations of magnetically arrested accretion flows and outflows around black holes, for toroidally and poloidally dominated magnetic fields (Tchekhovskoy et al. 2011). They furthermore demonstrated the possible extraction of net energy from a spinning black hole via the Penrose-Blandford-Znajek mechanism (McKinney et al. 2012). Moreover, there were radiation MHD/GRMHD simulations of accretion and outflows around black holes, exploring three distinct flow phases including the radiatively inefficient phase which is similar to the flows considered in the present work (Ohsuga & Mineshige 2011; McKinney et al. 2014). Some of the MHD simulations investigated the reasons behind the variability in low angular momentum, underluminous accretion flows in the vicinity of a supermassive black hole (Moscibrodzka et al. 2007). However, all these works, to the best of our knowledge, considered the cases when any viscosity to be arisen from magnetorotational instability (MRI) leading effectively to the Shakura-Sunyaev α viscosity (Balbus & Hawley 1991).

Here we plan to investigate, semi-analytically, the effects of large scale magnetic field, with plasma- $\beta > 1$ yet, on to the advective accretion flows in order to transport matter, however restricted to the simpler 1.5-dimensions. Therefore, we consider the flow variables, averaged over the vertical coordinate, to depend on the radial coordinate only. While the vertical equilibrium assumption corresponds to no vertical component of velocity, we choose the vertical component of magnetic field to be non-zero. Although, in reality, a non-zero vertical magnetic field induces a vertical motion, in the platform of the present assumption, any vertical motion will be featured as an outward motion. Nevertheless, whether it is a vertical or outward transport, our aim is to furnish removal of angular momentum from the flow via magnetic stresses, leading to the infall of matter towards black holes.

The plan of the paper is the following. In the next section, we describe the set of magnetohydrodynamic/hydromagnetic equations, at the limit of very large Reynolds number, as is the case in accretion disks, describing flow model. This is basically the set of Navier-Stokes equation, but in the presence of magnetic shearing stresses (and Lorentz force), magnetic induction equation, the condition for the absence of magnetic monopole, and finally the conservation of mass. Subsequently, we discuss the numerical solutions of the set of equations in §3 and its implications. Finally, we summarize the results along with a discussion in §4.

2. MODEL HYDROMAGNETIC EQUATIONS

We describe optically thin, magnetized, viscous, axisymmetric, advective, vertically averaged, steady-state accretion flow, in the pseudo-Newtonian framework with the Mukhopadhyay (2002) potential. The choice of the pseudo-Newtonian framework, for the present purpose, does not hinder any physics, compared to that would appear in the full general relativistic framework. Hence, the equation of continuity, vertically averaged hydromagnetic equations for energy-momentum balance in different directions are given by

$$\dot{M} = 4\pi x \rho h \vartheta, \quad (1)$$

$$\vartheta \frac{d\vartheta}{dx} + \frac{1}{\rho} \frac{dP}{dx} - \frac{\lambda^2}{x^3} + F = \frac{1}{4\pi\rho} \left(B_x \frac{dB_x}{dx} + s_1 \frac{B_z B_x}{h} - \frac{B_\phi^2}{x} \right), \quad (2)$$

$$\vartheta \frac{d\lambda}{dx} = \frac{1}{x\rho} \frac{d}{dx} (x^2 W_{x\phi}) + \frac{x}{4\pi\rho} \left(B_x \frac{dB_\phi}{dx} + s_2 \frac{B_z B_\phi}{h} + \frac{B_x B_\phi}{x} \right), \quad (3)$$

where $W_{x\phi} = \alpha(P + \rho\vartheta^2)$,

$$\frac{P}{\rho h} = \frac{Fh}{x} - \frac{1}{4\pi\rho} \left(B_x \frac{dB_z}{dx} + s_3 \frac{B_z^2}{h} \right), \quad (4)$$

$$\vartheta T \frac{ds}{dx} = \frac{\vartheta}{\Gamma_3 - 1} \left(\frac{dP}{dx} - \frac{\Gamma_1 P}{\rho} \frac{d\rho}{dx} \right) = Q^+ - Q^- = Q_{vis}^+ + Q_{mag}^+ - Q_{vis}^- - Q_{mag}^- \quad (5)$$

when we assume that the variables do not vary significantly in the vertical direction such that $\partial/\partial z \rightarrow s_i/z \sim s_i/h$, when $i = 1, 2, 3$, which is indeed true in the disk flows. Note that s_1, s_2 and s_3 are the degrees of scaling for the radial, azimuthal and vertical components of the magnetic field respectively. As a consequence, the vertical component of velocity is zero. Here \dot{M} is the conserved mass accretion rate and the corresponding equation is the integrated version of the continuity equation, ρ is the mass density of the flow, ϑ the radial velocity, P the total pressure including the magnetic contribution, F the force corresponding to the pseudo-Newtonian potential for rotating black holes, λ the angular momentum per unit mass, $W_{x\phi}$ the viscous shearing stress written following the Shakura-Sunyaev (Shakura & Sunyaev 1973) prescription with appropriate modification (Mukhopadhyay & Ghosh 2003), $h \sim z$, the half-thickness of the disk, s the entropy per unit volume, T the (ion) temperature of the flow, Q^+ and Q^- are the net rates of energy released and radiated out per unit volume in/from the flow respectively (when $Q_{vis}^+, Q_{mag}^+, Q_{vis}^-, Q_{mag}^-$ are the respective contributions from viscous and magnetic parts). We furthermore assume, for the present purpose, the heat radiated out proportional to the released rate with the proportionality constants $(1 - f_{vis})$ and $(1 - f_m)$, respectively, for viscous and magnetic parts of the radiations. Γ_1, Γ_3 indicate the polytropic indices depending on the gas and radiation content in the flow (see, e.g., Rajesh & Mukhopadhyay 2010a for exact expressions) and B_x, B_ϕ and B_z are the components of magnetic field. Note that, the independent variables x and z are the radial and vertical coordinates, respectively, of the flow, expressed in the units of GM/c^2 , where G is the Newton's gravitation constant, M the mass of the black hole and c the speed of light. Accordingly, all the above variables are made dimensionless, e.g. ϑ is expressed in the units of c . For any other details, e.g. model for Q_{vis}^+ , see the existing literature (Rajesh & Mukhopadhyay 2010a,b), when

$$Q_{vis}^+ - Q_{vis}^- = \frac{\alpha f_{vis}(P + \vartheta^2 \rho) \lambda}{x^2}. \quad (6)$$

We furthermore do not consider the heat generated and absorbed due to the nuclear reactions (Mukhopadhyay & Chakrabarti 2000, 2001). This is to emphasize that all the variables appearing in the equations are assumed to be their respective vertically averaged quantities.

Hydromagnetic flow equations must be supplemented by (for the present purpose, steady-state) equations of induction and no magnetic monopole, given by

$$\nabla \times \vec{v} \times \vec{B} + \nu_m \nabla^2 \vec{B} = 0, \quad (7)$$

$$\frac{d}{dx} (x B_x) + s_3 \frac{B_z}{h} = 0, \quad (8)$$

when \vec{v} and \vec{B} are respectively the velocity and magnetic field vectors and ν_m is the magnetic diffusivity. On taking the ratio of the orders of the first to the second (diffusive) terms in equation (7), we obtain $L|\vec{v}|/\nu_m = R_m$, when L being the order of the length scale of the system. Hence, when the Reynolds number, R_m , is very large, which is the case for accretion disks, the second term (which is associated with the magnetic diffusivity) in equation (7) can be neglected. However, this term can be rather important inside some localized regions in certain astrophysical systems due to subtle reasons. Nevertheless, for the present purpose, for simplicity we will neglect this term throughout. Furthermore, as ϑ and λ are assumed to be independent of the vertical coordinate, it is easy to check from the radial component of equation (7) that $\partial B_z / \partial z \rightarrow 0$ (and hence $B_z/h \rightarrow 0$). Therefore, the azimuthal and vertical components of equation (7), at large R_m , respectively lead to

$$\frac{d}{dx} \left(\vartheta B_\phi - \frac{B_x \lambda}{x} \right) = 0, \quad (9)$$

$$\frac{d}{dx}(x\vartheta B_z) = 0, \quad (10)$$

when the radial component of equation (7) turns out to be trivial. Subsequently the equation (8) reduces to

$$\frac{d}{dx}(xB_x) = 0. \quad (11)$$

Because of the choice of very large R_m (ideal MHD), there is a perfect flux freezing in the flow. Therefore, a steady advection of the vertical magnetic flux towards the center may lead to the decrease of β , making it close to unity and further smaller, in a pure axisymmetric flow, even if the initial β was high. Hence, at some point, the back reaction of the field will inhibit accretion, depending on the geometry of the field lines. Although the physics of this process is not captured by the equations above and we also do not intend to discuss such physics, we will show below in §3 the effects of higher magnetic fields, in particular the inner edge (around the critical radius), in the entire flow structure. This is essentially to capture a situation after significant advection done with a certain field geometry.

Henceforth, we will also neglect the second term in the parenthesis of equation (4). The set of equations (1), (2), (3), (4), (5), (9), (10) and (11) is essentially the modified version of the set of advective accretion disk equations in the presence of a large-scale magnetic effect, which are otherwise discussed in the literature in absence of it. Generally, in order to understand the hard state of accretion flows around black holes, the flow is assumed to be purely hydrodynamic with the consideration of a turbulent viscosity arisen due to a weak magnetic field, i.e. MRI (Balbus & Hawley 1991). Although, MRI is a largely accepted idea, so far, in order to explain the origin of turbulence in accretion disks, there are some subtle issues with it (e.g. Mahajan & Krishan 2008; Mukhopadhyay et al. 2011) including its applicability in colder disks. Therefore, transport of matter in disks is much more transparent through magnetic stresses, if the flows are embedded with a large scale field. Such magnetic stresses are considered here on the right hand side of the radial, azimuthal and vertical momentum balance equations. In addition, the magnetic heating due to abundant supply of magnetic energy and the annihilation of the magnetic fields (Bisnovatyi-Kogan & Ruzmaikin 1974; Choudhuri 1998), which effects may be small however, is considered in the energy equation such that

$$Q_{mag}^+ - Q_{mag}^- = \frac{3f_m|\vec{B}|^2\vartheta}{16\pi x}. \quad (12)$$

However, the other related terms in the momentum balance equations are neglected, again in comparison with the remaining terms in the respective equations, for the purpose of the present work.

Therefore, even in the absence of turbulent viscosity ($\alpha = 0$) and hence viscous stresses, magnetic stresses alone can help in transporting matter in the accretion flows. Such a consideration of large scale magnetic field and hence transport via magnetic stress has, although been considered in circumstellar disks around young stars (see Königl & Salmeron 2011 for a recent review), not yet been considered for advective accretion flows around black holes.

Question may arise, if the magnetic field with plasma- $\beta > 1$ is adequate enough to describe infall of matter in order to explain observation. We will show in the next section that the large-scale magnetic field, even with a significantly large plasma- β , can describe advective accretion flows as efficiently as an α does.

2.1. Solution procedure

We have seven equations (excluding the vertical momentum balance equation, which assures the vertical magnetostatic balance) and seven variables: $\vartheta, \lambda, P, \rho, B_x, B_\phi, B_z$, which we plan to solve along with the vertical magnetostatic balance condition. First, we plan to reduce $d\vartheta/dx$ in terms of other variables and the independent variable x alone (without any other derivatives), given by

$$\frac{d\vartheta}{dx} = \frac{\frac{1}{F}\frac{dF}{dx} - \frac{3}{2x} - \frac{\rho}{2P}\left(1 + \frac{1}{\Gamma_1}\right)\left(\frac{\lambda^2}{x^3} - F + \frac{1}{4\pi\rho}\left[\frac{s1B_zB_x}{h} - \frac{B_\phi^2 + B_x^2}{x}\right]\right) + \frac{\Gamma_3 - 1}{2\vartheta\Gamma_1 P}\left[\frac{\alpha_{vis}(P + \vartheta^2\rho)\lambda}{x^2} + \frac{3f_m|\vec{B}|^2\vartheta}{16\pi x}\right]}{\frac{1}{\vartheta} - \frac{\vartheta\rho}{2P}\left(1 + \frac{1}{\Gamma_1}\right)}. \quad (13)$$

As the advective accretion around black holes is necessarily transonic, the flow must pass through a critical radius where

$$\vartheta = \vartheta_c = \sqrt{\frac{2\Gamma_1 P_c}{\rho_c(1 + \Gamma_1)}}, \quad (14)$$

when the variables with subscript ‘c’ indicate the respective values at that critical radius and the numerator of equation (13) has to be zero for a continuous solution. At the critical radius, we also prescribe

$$B_{xc} = B_{yc} = B_{zc} = \sqrt{4\pi\rho_c} \frac{c_{sc}}{f_A\sqrt{3}}, \quad \text{when } c_s = \text{sound speed} \sim \sqrt{\frac{P}{\rho}}, \quad (15)$$

so that the Alfvén velocity is a fraction of sound speed therein. Although this is a simpler prescription, other choices do not change the picture, being addressed in this work, qualitatively. Note that a steady MHD flow would normally

have three critical points – fast magnetosonic point, Alfvén point and slow magnetosonic point – of which the Alfvén point is not a true critical point (Gammie 1999). The remaining two physically important distinct critical points, corresponding to fast and slow magnetosonic waves, collapse into a single point because of the assumptions made in equation (15). We typically choose $f_A \sim 10 - 10^3$ in our various computations (see the figures). This is to capture a situation when magnetic pressure at the inner edge of the flow is not high enough to hinder radial infall of the matter, i.e. a situation with a weak back reaction of magnetic fields. Furthermore, from equation (11), we can write

$$x_c B_{xc} = \text{constant} = C_0 = x B_x \quad (16)$$

which fixes the profile of B_x throughout the flow.

These four conditions, along with the conditions that $\lambda = \lambda_K$ (when λ_K being the Keplerian angular momentum per unit mass) and $\vartheta \ll 1$ at the beginning of sub-Keplerian flow far away from the black hole, i.e. outer boundary, and $\vartheta \sim 1$ at $r \sim r_+$ serve as important boundary conditions. Based on all the conditions, by solving the set of seven coupled differential equations, we can obtain the profiles for all variables including those of B_y and B_z . Of course, then, one has to supply M , \dot{M} , α , γ and hence f_{vis} and f_m for a flow. See Rajesh & Mukhopadhyay (2010a) for the solution procedure in details.

3. SOLUTIONS

Our main aim is to obtain the solutions of magnetized accretion flows. In other words, the aim is to understand how the large scale magnetic field (alone) can influence the mass transfer in accretion process, in particular in the the advective regime. This is important in the light of our ignorance of the origin of viscosity in accretion flows, which may be arisen from turbulence, as the molecular viscosity therein is well-known to be insignificant. Hence, our venture here is to investigate, if the large-scale magnetic field, however with large plasma- β , can govern the same/similar transport of angular momentum as the well-known α -prescription does. Hence, we plan to understand the relative strengths between the magnetic stress tensors and the viscous stress tensors in order to control advective accretion flows.

Our plan is to explore specifically two situations. (1) Flows with a relatively higher \dot{M} and, hence, lower γ , modelled around stellar mass black holes: such flows may or may not form Keplerian accretion disks. (2) Flows with a lower \dot{M} and, hence, higher γ , modelled around supermassive black holes: such flows are necessarily hot gas dominated advective (or advection dominated) accretion flows.

3.1. Accretion around stellar mass black holes

We choose $M = 10M_\odot$ and $\dot{M} = 0.1\dot{M}_{Edd}$, when M_\odot and \dot{M}_{Edd} are the solar mass and the Eddington accretion rate respectively. However, this choice of \dot{M} does not necessarily imply the flow to be purely Keplerian, rather advective, which is indeed, in general, under consideration. Such flows have temperature $T \gtrsim 10^9\text{K}$ and $\rho \gtrsim 10^{-7}\text{gm/cc}$ (Sinha et al. 2009; Rajesh & Mukhopadhyay 2010a), which were extensively explored in the context of the formation of shock in hot accretion disks and subsequent outflows and observed spectral states (Chakrabarti & Titarchuk 1995; Chakrabarti 1996). A relation of γ (and \dot{M}) with the ratio of the gas and radiation content of the flow and the corresponding variations have been discussed by Mukhopadhyay & Dutta (2012). Therefore, following previous authors and for the convenience of comparison of the previous results without magnetic fields, we choose $\gamma = 1.335$ along with the intermediate f_{vis} and f_m . Note that while the results depend on the sign of s_2 , they do not depend on s_1 and s_3 .

Figure 1 compares accretion flows (1) in the presence of large scale magnetic field, but in the absence of α -viscosity: magnetic flow, and (2) in the presence of viscosity, but in the absence of large scale magnetic field: viscous flow. Here we consider B_ϕ to have the same direction as of λ . It shows that the large scale magnetic field $\sim 10^4 - 10^5\text{G}$, with its distribution in the inner edge of the flow defined by equation (15), is adequately able to transport angular momentum, as viscous flows do with $\alpha = 0.017$ and 0.012 respectively for nonrotating and rotating black holes. Figures 1a,b show that the disk sizes are the same for the respective viscous and magnetic sub-Keplerian flows with the above mentioned respective α -s. However, the transport of angular momentum takes place faster in magnetic flows, where the flows become quasi-spherical at larger radii than at the radii they become in the respective viscous flows. Away from the black hole, B_ϕ increases, which implies that the matter is prone to outflow from the outer region through the magnetic field lines extended outward direction. In a self-consistent model, including the flow variation in the vertical direction, the above features should have been appeared as the increasing magnetic field with the vertical coordinate. Such a model is planned to develop in future. In the present 1.5-dimensional magnetic flow model, when the magnetic stresses play the main role to remove angular momentum and hence to overcome the centrifugal barrier, as matter advances towards the black hole, the magnetic field decreases. Note that, as shown in Figs. 1b,d, λ and $|B_\phi|$ both decrease towards the black hole: to overcome large λ , the flow needs a large $|B_\phi|$ and vice versa — they are the self-consistent solutions to each other. Note furthermore that the negative sign in B_ϕ in Fig. 1(d) is due to our choice of decreasing B_ϕ with increasing z , i.e. negative s_2 , in the outer edge of the flow (which results in the same trend of the flow almost throughout, except very inner region). Figure 1(c) shows that our chosen regime of magnetic flows, allowing a steady infall of matter, corresponds to a relatively high plasma- β (actually inverse of β in shown). This furthermore renders lower magnetic pressures in respective flows compared to their maximum allowed values based on the virial theorem/equipartition principle. As discussed in the previous works (e.g. Mukhopadhyay 2003), a strong centrifugal effect is depicted in the Mach number profiles in Fig. 1a (featured as slowing down the matter at around $x = 20 - 25$) in the high angular momentum flows around nonrotating black holes, compared to the low angular momentum flows around rotating black holes.

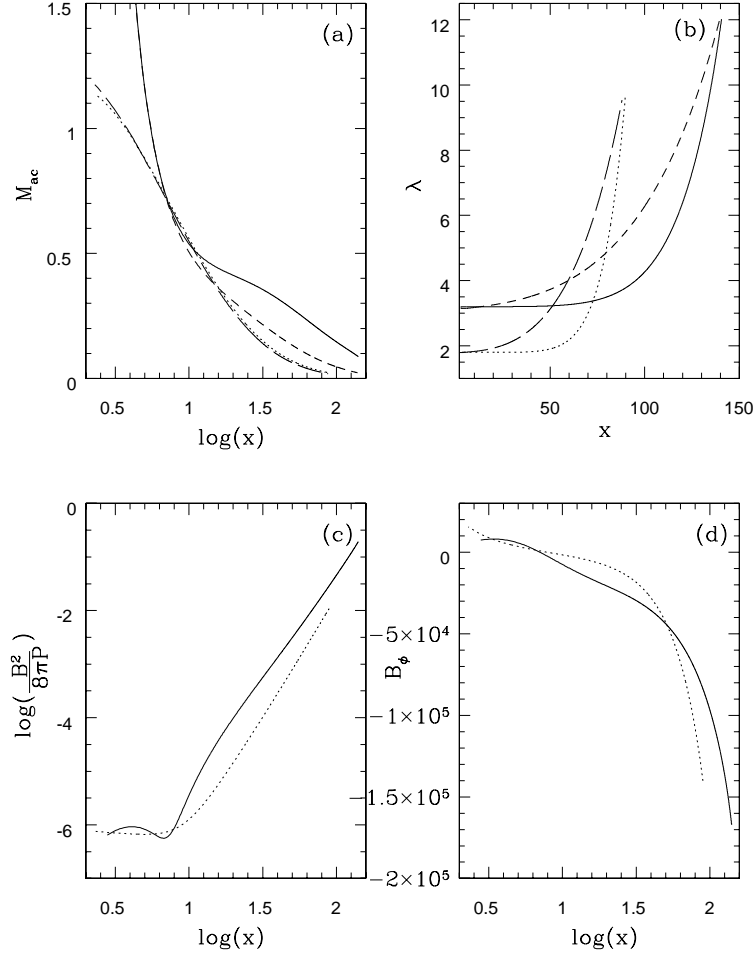


FIG. 1.— (a) Mach number, (b) angular momentum per unit mass in GM/c , (c) inverse of plasma- β , (d) azimuthal component of magnetic field in Gauss, when solid and dotted lines are for magnetic flows around Schwarzschild ($a = 0, \lambda_c = 3.2$) and Kerr ($a = 0.998, \lambda_c = 1.8$) black holes respectively, and dashed and long-dashed lines are for viscous flows around Schwarzschild ($a = 0, \alpha = 0.017, \lambda_c = 3.15$) and Kerr ($a = 0.998, \alpha = 0.012, \lambda_c = 1.8$) black holes respectively. Other parameters are $M = 10M_\odot$, $\dot{M} = 0.1\dot{M}_{Edd}$, $\gamma = 1.335$, $f_{vis} = f_m = 0.5$, $s_2 = -0.5$.

Now we hypothesize that B_ϕ increases with the increasing z , i.e. positive s_2 , at the outer edge of the flow (which results in the same trend of the flow almost throughout, except very inner region). In this case, the right hand side of equation (3), for a magnetic flow, completely flips sign compared to the negative s_2 case. Figure 2 shows that as the matter infalls, the toroidal component of magnetic field slows down the azimuthal motion of matter faster, making $\lambda = 0$ and, subsequently, inverting the orientation of λ . This is effectively due to the change in signs of magnetic stress components: $B_x B_\phi$ and $B_z B_\phi$. Figure 2b shows the variation of the magnitude of λ , as the matter falls in. λ is positive far away from the black hole, but it is negative close to the black hole. The location around $\lambda = 0$ reveals a trough-like region in the flow. Hence, in the either sides of $\lambda = 0$, there is a stronger centrifugal barrier which stores matter around $\lambda = 0$ (due to the competition between radial and azimuthal flows). This region is prone to kick the matter out, producing outflows. Hence, if B_ϕ increases with z in the flow to start with, then as matter advances towards the black hole, a “potential well” forms to produce outflows. Note, however, that very close to a rotating black hole, matter will be under the influence of the black hole completely and hence λ cannot have an opposite sign with respect to that of the black hole. Therefore, this solution is not valid very close to the rotating black hole. Indeed, the pseudo-Newtonian description is not applicable very close to the black hole. Nevertheless, the above solution implies a possibility of having such an origin of outflows in a magnetized accretion flow in the presence of a finite conductivity (when the field is not frozen with the matter, when the term associated with magnetic diffusivity in the induction equation is retained). Note that the plasma- $\beta > 1$ is maintained throughout the flows.

This furthermore motivates us to check with such a possibility in viscous flows with viscosities $\alpha = 0.08$ and 0.056 respectively for nonrotating and rotating black holes. As shown in Figs. 2a,b, the angular momentum profiles in the respective magnetic and viscous flows appear to be similar, which furthermore makes the respective radial velocity profiles similar, unlike the previous cases, as shown in Fig. 1, with the positive λ throughout. In the previous cases, the magnetic stresses are able to remove the angular momentum faster than the viscous stresses, in particular at a large distance from a Schwarzschild black hole, which is clearly understood from Figs. 1a,b. Although the same is true

for a Kerr black hole, as the disk angular momentum itself is lower there, it does not effectively create any impact on the Mach number profiles. However, due to the choice of larger α , in the counterrotating cases, viscous stresses appear to be almost equivalent to the magnetic stresses and hence the radial velocity profiles in either of the respective flows appear similar.

Let us now understand in more details, how the various components of magnetic stress are responsible for inflow and/or angular momentum transfer therein. Figures 3a,b show the variations of various components of the magnetic field as functions of the radial coordinate, around Schwarzschild and Kerr black holes, which are responsible for the various components of magnetic stress tensor. The profiles of field components and their magnitudes are partly dependent on their prescription given by equation (15). Figure 4a shows that the stress tensor component $B_x B_z$ around a Schwarzschild black hole increases almost throughout as matter advances towards the black hole. This implies that the flow is prone to outflow through the field lines, which indirectly helps in removing the angular momentum, which furthermore renders its infall towards the black hole. However, very close to the black hole, $B_x B_z$ decreases, as indeed outflow is not possible in the near vicinity of the black hole, in particular, once the matter passes through the (inner) sonic point. By this radius, the flow angular momentum becomes very small which practically does not affect the infall. The magnitude of $B_\phi B_z$ decreases till the inner region of accretion flow, implying the matter to be spiralling out and hence removing the angular momentum. A larger $|B_\phi B_z|$ at a larger radius implies a requirement of the removal of larger λ therein. This automatically emerges from the self-consistent solutions of the set of equations. Nevertheless, close to the black hole, this effect reverses, rendering infall. Finally, the magnitude of $B_x B_\phi$ increases at a large and a small distances from the black hole (except around the transition radius), which helps infall, in the same way as the Shakura-Sunyaev viscous stress would do with the increase of matter pressure. However, at the intermediate zone, the transfer of angular momentum through $B_x B_\phi$ reverses and a part of the matter outflows. At the Keplerian to sub-Keplerian transition zone, flow/disk thickness increases, which effectively kicks the matter vertically, showing a decrease of $B_x B_\phi$. Most of these features remain unchanged for the flow around a rotating black hole, as shown in Fig. 4b. However, as a rotating black hole renders a stronger/efficient outflow/jet, here, except at the inner zone, $B_x B_\phi$ decreases throughout, which helps transferring the angular momentum inwards and kicking the matter outwards. Nevertheless, such a flow does not exhibit a high λ either so that does not necessarily require an increasing $B_x B_\phi$ to remove λ . Figures 4c,d furthermore confirm that the above properties are invariant for the cases of B_ϕ increased with increasing z . The only difference here is that $B_x B_\phi$ and $B_\phi B_z$ have opposite signs with respect to the previous cases, when the components of magnetic field considered here are their respective averaged values. Note that all the components of magnetic stress tensor as functions of radius are determined by the associated components of magnetic field. The components of magnetic field are, however, determined by solving the underlying set of equations self-consistently with their prescription at the inner edge of the flow for the regime of interest.

Figure 5 compares three cases of magnetic flows. (1) A counter rotating disk throughout, when B_ϕ decreases with z almost throughout (solid line). (2) A disk having B_ϕ increasing with z almost throughout, which is corotating far away, but counterrotating close to the black hole (dotted line). (3) A disk having B_ϕ increasing with z almost throughout, which is counterrotating far away, but corotating close to the black hole (dashed line). Note importantly in the latter two cases that the increasing B_ϕ with disk height induces the change of handedness of the disk during the infall of matter. However, the profiles of Mach number and β practically appear similar in all the three cases.

In Fig. 6, we compare the disk hydrodynamics between the magnetic flows with high and low magnetic fields. As expected, a flow with the higher magnetic field transports the angular momentum much faster, leading to a smaller sub-Keplerian flow. In other words, in the presence of a higher magnetic field, when the magnetic stresses are stronger, the Keplerian flow (when $\lambda = \lambda_K$) as well as the boundary between the Keplerian and sub-Keplerian flows are able to advance towards the black hole, shrinking the sub-Keplerian zone because of efficient angular momentum transfer. Figure 6c shows that at a given radius the magnetic pressure, and hence the Alfvén speed, is much larger in a flow with the higher magnetic field (but still $\beta > 1$). Naturally, such a high field magnetic flow would be equivalent to a viscous flow with much larger α , compared to the cases shown in Fig. 1. An even higher magnetic field in the inner region would hinder any infall due to backreactions. Interestingly, in the radii close to the black hole, the sign of B_ϕ becomes distinctly opposite than the outer region in the high magnetic field case. However, by this radius the required amount of angular momentum has already been transferred outwards in order to advance the matter close to the black hole and hence the change in sign of B_ϕ does not create any physical impact onto the flow. If we vary the conditions chosen in equation (15), e.g., assume the components of magnetic field unequal, the qualitative picture remains unchanged — the magnetic stress in the presence of a large scale magnetic field could adequately transfer angular momentum. However, it is very important to note that if the strength of magnetic field and the corresponding value at the inner edge around the sonic radius would have been even higher, above a certain value, then the infall would no longer be possible. This is similar to the situation when above a certain value of λ at the sonic radius in a given flow, the infall is no longer possible (see, Rajesh & Mukhopadhyay 2010a).

If the flow is gas pressure dominated with larger γ , then all the above basic features remain the same. Be it radiation or gas dominated, large scale magnetic stresses, yet $\beta > 1$, can transport angular momentum as efficiently as the α -prescription does. Nevertheless, below we discuss the effects of large scale magnetic field in the gas pressure dominated flows around a supermassive black hole.

3.2. Gas dominated accretion around supermassive black holes

The supermassive black hole at the centre of our galaxy, Sgr A*, presumably exhibits a gas pressure dominated, advection dominated, accretion flow. This motivates us to undertake this case, when we choose $M = 10^7 M_\odot$ and

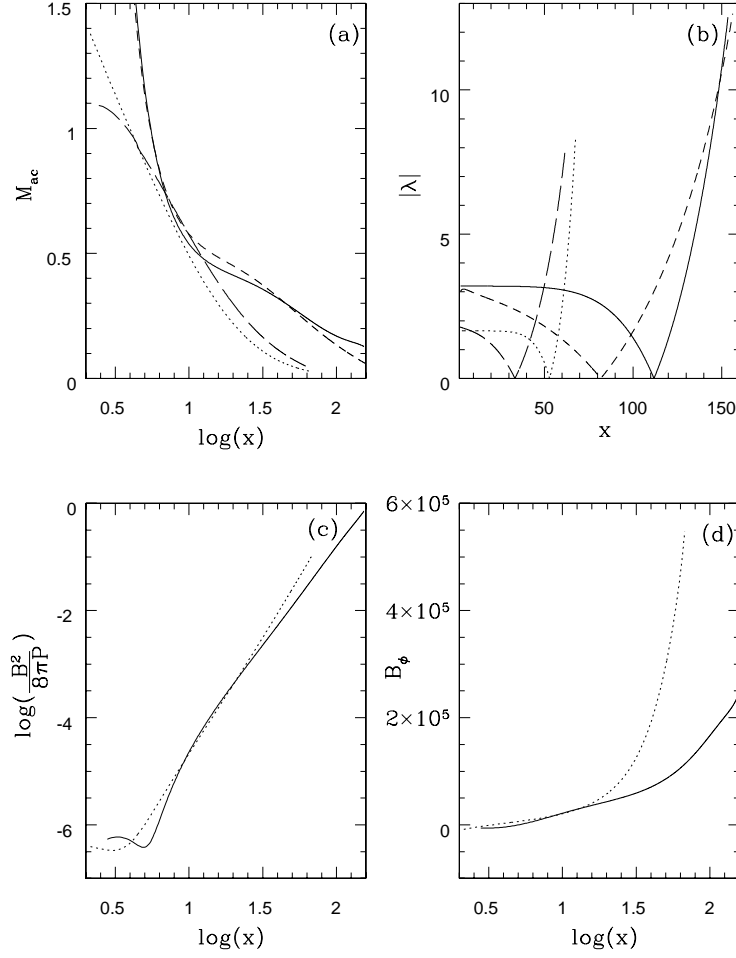


FIG. 2.— Same as in Fig. 1, except $s_2 = 0.5$, when $\lambda_c = -3.2$ and -1.65 for nonrotating and rotating magnetic flows respectively and $\alpha = 0.08$, $\lambda_c = -3.08$ and $\alpha = 0.056$, $\lambda_c = -1.74$ for nonrotating and rotating viscous flows respectively.

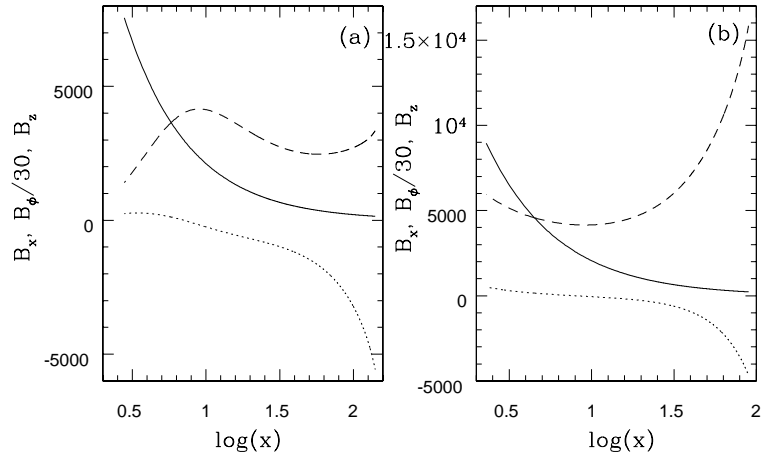


FIG. 3.— Components of magnetic field: B_x (solid line), B_ϕ but normalized by 30 (dotted line), B_z (dashed line), for (a) Schwarzschild magnetic flow of Fig. 1, (b) Kerr magnetic flow of Fig. 1.

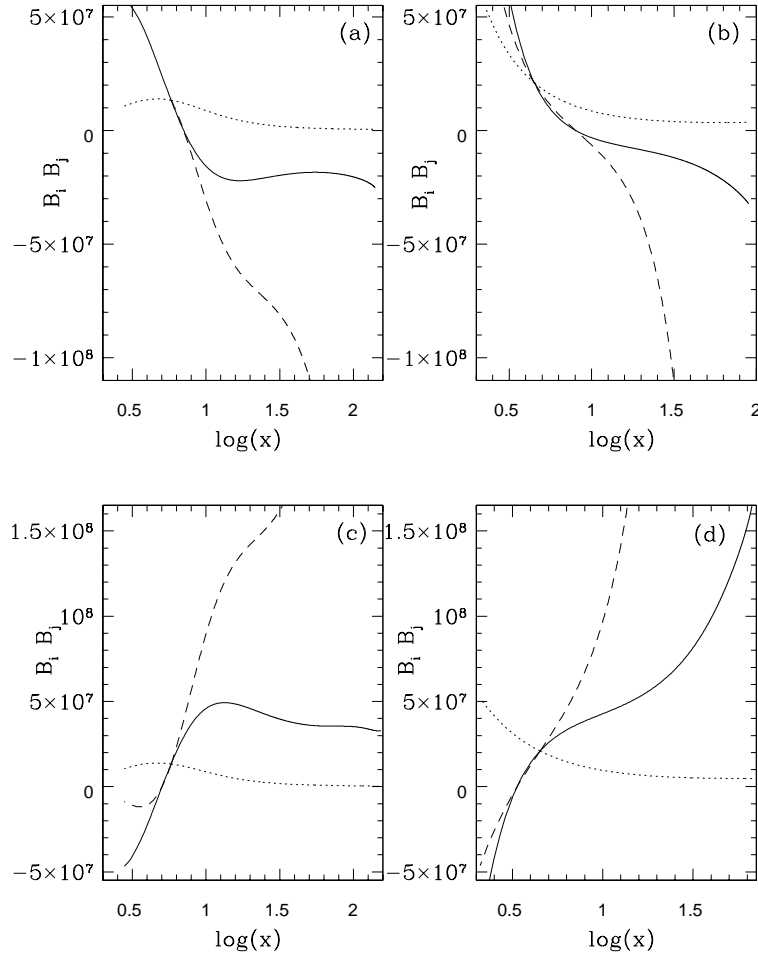


FIG. 4.— Components of magnetic stress: $B_x B_\phi$ (solid line), $B_x B_z$ (dotted line), $B_\phi B_z$ (dashed line), for (a) Schwarzschild magnetic flow of Fig. 1, (b) Kerr magnetic flow of Fig. 1, (c) Schwarzschild magnetic flow of Fig. 2, (d) Kerr magnetic flow of Fig. 2.

$M = 10^{-4} M_{Edd}$, and hence an appropriate $\gamma = 1.55$. We furthermore choose $f_{vis} = f_m = 0.95$, when strongly advective matter hardly has a chance to radiate photons. However, such a flow may also arise around a stellar mass black hole, e.g. microquasars.

The basic features in Fig. 7 are similar as those for the stellar mass cases, as shown in Fig. 1. However, due to the gas dominance, and hence lower angular momenta, the Mach number profiles practically do not have any centrifugal barrier. Such flows are hotter, with $T \gtrsim 10^{11}$ K, and more quasi-spherical, compared to the radiation dominated flows, when a very small part of the dissipated heat can be radiated away. However, the most significant difference in these flows lies in their low magnetic fields, compared to those discussed in §3.1. This is due to the largeness of black hole masses in these flows, which leads to a much larger size of sub-Keplerian flows, when the dimensional flow size scales as M . As a result, due to the law of equipartition, the magnetic field decreases significantly compared to the cases of stellar mass black hole, as shown in Fig. 7d (as compared to those shown in Fig. 1d).

The magnetic flows around nonrotating and rotating, both the black holes, have their viscous counterparts with respective $\alpha = 0.011$ and 0.0092 . This furthermore confirms that even in gas dominated flows, the large scale magnetic field is able to transfer the angular momentum as efficiently as the α -prescription does with the most plausible values of α .

Hypothesizing the increasing B_ϕ with increasing z at the outer edge of the sub-Keplerian flow, we obtain the same results as those for stellar mass black hole accretion flows described above, except at much lower magnetic fields. Like the stellar mass cases, here also a “potential well” forms which is featured in Fig. 8b, rendering the systems to have a zone for producing outflows. We do not repeat the detailed properties of it. Figures 8a,b also show the viscous flows resembling magnetic flows with $\alpha = 0.075$ and 0.07 respectively for nonrotating and rotating black holes.

3.3. Dependence on s_1, s_2, s_3

As defined in §2, s_1, s_2, s_3 parametrize the scaling of the variations of B_x, B_ϕ, B_z respectively in the vertical direction. This has to be considered because of our averaging the flows in the vertical direction, while the variation of magnetic field in the vertical direction has not been neglected, as has not been for P . Interestingly, the solutions practically do not depend on the choices of s_1 and s_3 . However, with the increase of the magnitude of s_2 , which implies the increasing

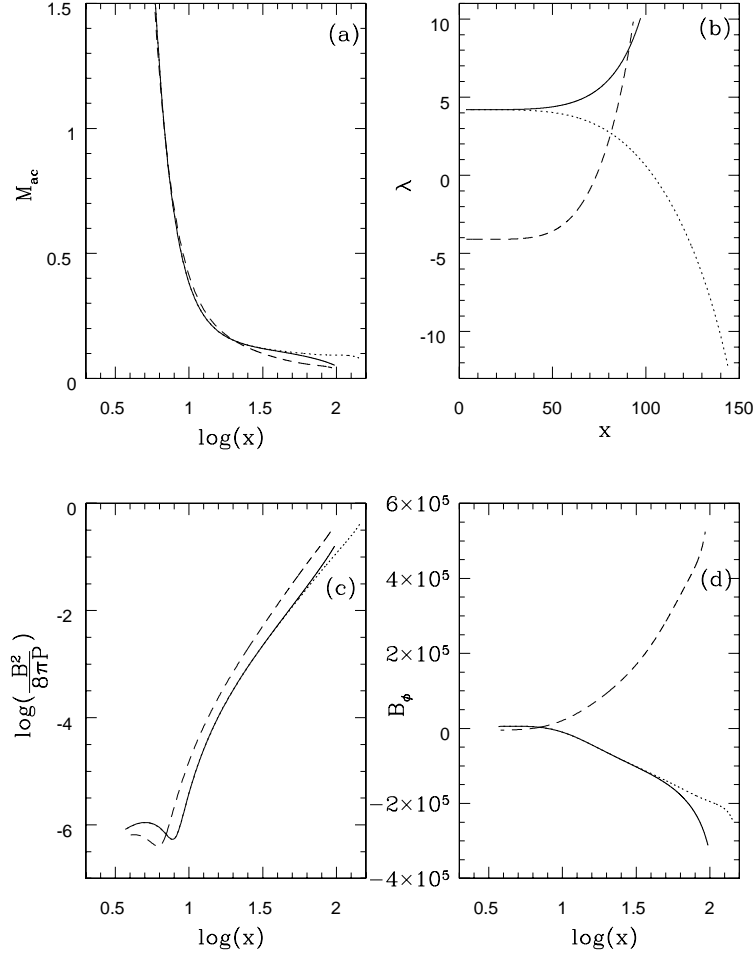


FIG. 5.— Same as in Fig. 1, but all for magnetic flows around a black hole with $a = -0.998$, when the solid line is for $s_2 = -0.5, \lambda_c = 4.2$ and the dotted and dashed lines are for $s_2 = 0.5$ with $\lambda_c = 4.2$ and -4.1 respectively.

change of magnetic field with the vertical coordinate, the size of sub-Keplerian flow decreases. This is because, stronger the vertical variation of magnetic field, larger the change of the magnetic stresses in the vertical direction, on average faster the infall of matter is. This also argues for the faster rate of throwing the matter via outflows, when the outflows, in a more self-consistent 2.5-dimensional flow, are expected to plunge out via the magnetic field lines in the vertical direction. Hence, with the increase of magnetic field in the vertical direction, the system becomes more prone to outflow matter. Subsequently, a faster rate of outflow renders a faster removal of angular momentum and hence a faster rate of infall. As a result, the flow could remain Keplerian with the aid of adequate mechanisms of angular momentum transfer, till further inner region of the flow. Hence, the boundary between the Keplerian and sub-Keplerian flows advances towards the black hole.

3.4. Interconnection between advection and magnetic field

As the current \vec{J} in the conducting fluid with conductivity σ and electric field \vec{E} is known to be $\vec{J} = \sigma (\vec{E} + \vec{v} \times \vec{B})$, Faraday's law of induction in the steady-state for axisymmetric accretion disks considered here, as given by equation (7), can be recalled as

$$\nabla \times \frac{\vec{J}}{\sigma} - \nabla \times (\vec{v} \times \vec{B}) = 0. \quad (17)$$

For a flow with very large R_m (when $\nu_m \propto \sigma^{-1} \ll 1$), the z -component of equation (17), averaged in z and integrated over ϕ , is given by

$$\int \vartheta B_z x d\phi = \text{constant} = C, \quad (18)$$

when the constant can be identified as $C = d/dt (\int B_z ds_{x\phi}) = d\Phi/dt$, where $ds_{x\phi}$ is the elementary surface area in the disk plane and Φ is the magnetic flux. Therefore, equation (18) fixes the relation between advection and B_z , and

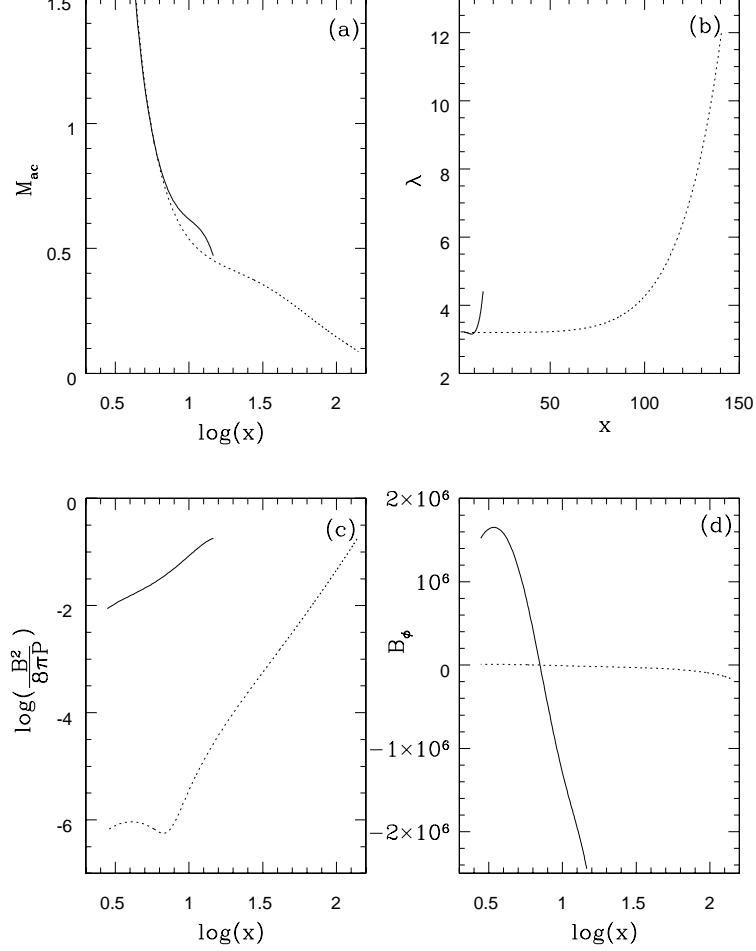


FIG. 6.— Same as in Fig. 1, but comparing between the flows with high (solid line) and low (dotted line) magnetic fields around a nonrotating black hole with $s_2 = -0.5$.

hence the magnetic flux in the accretion flow. This also can be understood by recasting Faraday's law of induction into

$$\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0, \quad \text{when} \quad \vec{B} = \nabla \times \vec{A}, \quad (19)$$

which furthermore argues for

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} - \vec{C}, \quad (20)$$

when V is the Coulomb potential and \vec{C} is a constant vector. Hence, for a steady axisymmetric accretion flow

$$E_\phi = \frac{J_\phi}{\sigma} + x\vartheta B_z = -C_\phi. \quad (21)$$

Therefore for $R_m \gg 1$, $x\vartheta B_z$ is conserved. The constant C_ϕ or C can be fixed from a given boundary condition. In really, however, the flow is not expected to be purely axisymmetric and, hence, C_ϕ or C can also be mimicked as the contribution from non-axisymmetry. Earlier Lubow et al. (1994) discussed a model of geometrically thin accretion flows in the presence of weak magnetic field, but assuming $C_\phi = 0$ which is not true in general.

The constraint on advection arisen in equation (21) is clearly visible in the velocity profiles in Fig. 1a with respect to the variations of the vertical component of magnetic field shown in Fig. 3. For the magnetic flow around a Schwarzschild black hole, ϑ first increases steadily with the decrease of B_z at larger radii and subsequently matter tends to slow down due to centrifugal effect (with the relative decrease of infall rate $d\vartheta/dx$) with the increase of B_z until $x = 10$. Finally, matter plunges into the black hole steadily with a sharp decrease of B_z . For a rotating black hole, however, ϑ steadily increases with the steady decrease of B_z almost throughout. Nevertheless, very close to the black hole, B_z slightly increases due to the spin effect of black hole, decreasing $d\vartheta/dx$ slightly. This hints the power of black hole's spin to plunge the matter out.

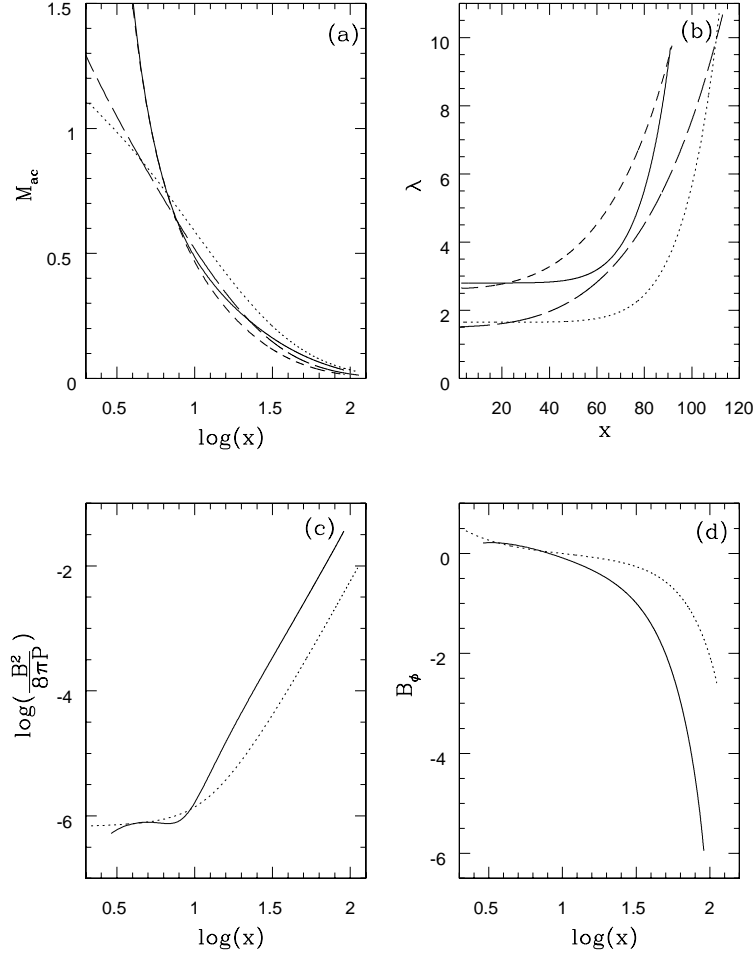


FIG. 7.— (a) Mach number, (b) angular momentum per unit mass in GM/c , (c) inverse of plasma- β , (d) azimuthal component of magnetic field in Gauss, when solid and dotted lines are for magnetic flows around Schwarzschild ($a = 0, \lambda_c = 2.8$) and Kerr ($a = 0.998, \lambda_c = 1.65$) black holes respectively, and dashed and long-dashed lines are for viscous flows around Schwarzschild ($a = 0, \alpha = 0.011, \lambda_c = 2.65$) and Kerr ($a = 0.998, \alpha = 0.0092, \lambda_c = 1.52$) black holes respectively. Other parameters are $M = 10^7 M_\odot$, $\dot{M} = 10^{-4} \dot{M}_{Edd}$, $\gamma = 1.55$, $f_{vis} = f_m = 0.95$, $s_2 = -0.5$.

4. DISCUSSION AND SUMMARY

We have discussed the power of large scale magnetic field in advective accretion flows around black holes in order to transport angular momentum, enabling infall of matter. In a simpler Keplerian, self-similar model framework, such an investigation had been initiated by Blandford & Payne (1982) long ago, and in the cases of circumstellar disks around young stars (e.g. Königl & Salmeron 2011), such an approach has been explored. However, it remained unexplored, to the best of our knowledge, in the advective accretion disk around black holes, when it may exhibit a hard spectral state, until this work. Note that, often, only hard spectral states of disks are associated with the outflows/jets.

We have found that the flows with plasma- $\beta > 1$ exhibit adequate magnetic transport — as efficient as the α -viscosity with $\alpha = 0.08$ would do. This is interesting as the origin of α (and the corresponding instability and turbulence) is itself not well understood. The maximum required large scale magnetic field is a few factor times 10^5G in a disk around $10 M_\odot$ black holes and $\sim 10 \text{G}$ in a disk around $10^7 M_\odot$ supermassive black holes. The presence of such a field, in particular for a stellar mass black hole disk when the binary companion supplying mass is a Sun-like star with the magnetic field on average 1G , may be understood, if the field is approximately frozen with the disk fluids (or the supplied fluids from the companion star remain approximately frozen with the magnetic field) or disk fluids exhibit large Reynolds number. Indeed, all the present computations are done at the limit of large Reynolds number, as really is the case in accretion flows, such that the term associated with the magnetic diffusivity in the induction equation can be neglected. The size of a disk around supermassive black holes is proportionately larger compared to that around a stellar mass black hole. Hence, from the equipartition theory, indeed the magnetic field is expected to be decreased here compared to that around stellar mass black holes.

Is there any observational support for the existence of such a magnetic field, as required for the magnetic accretion flows discussed here? Interestingly, the polarization measurements in the hard state of Cyg X-1 imply that it should have at least 10mG field at the source of emission (Laurent et al. 2011). In order to explain such high polarization, a jet model was suggested by Zdziarski et al. (2014), which requires a magnetic field $\sim (5 - 10) \times 10^5 \text{G}$ at the base of

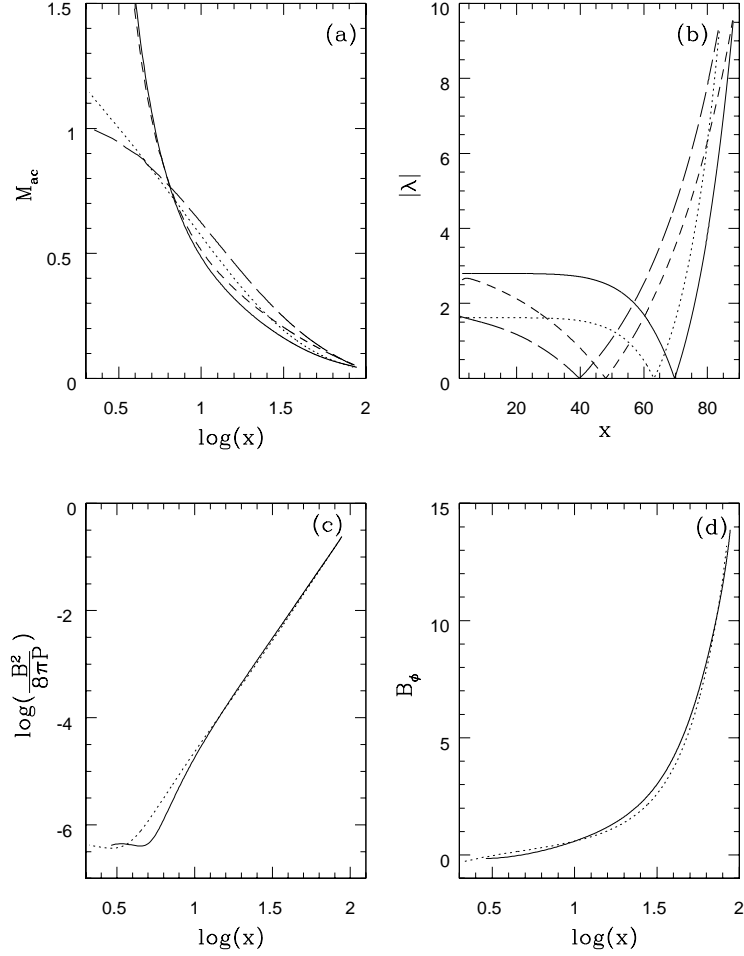


FIG. 8.— Same as in Fig. 7, except $s_2 = 0.5$, when $\lambda_c = -2.8$ and -1.62 for nonrotating and rotating magnetic flows respectively and $\alpha = 0.075$, $\lambda_c = -2.65$ and $\alpha = 0.07$, $\lambda_c = -1.6$ for nonrotating and rotating viscous flows respectively.

jet and hence in the underlying accretion disk. Also, the magnetic field in the inner region of accretion disks around more than a dozen black holes has been found to be very high, based on a model relating the observed kinetic power of relativistic jet to the magnetic field of accretion disks (Piotrovich et al. 2014).

Different components of magnetic stress tensor have different roles: $B_x B_\phi$ controls the infall in the disk plane, whereas $B_\phi B_z$ renders the flow to spiral outwards and, hence, outflow. Moreover, $B_x B_z$ helps to kick the matter out vertically. Larger the field strength, larger is the power of magnetic stresses. Interestingly, the magnitude of magnetic field decreases, as the steady-state matter advances towards the black hole. This is primarily because $B_\phi B_z$ (and also $B_x B_\phi$ for a rotating black hole) decreases inwards almost entirely in order to induce outflow. This furthermore reveals a decreasing $|B_\phi|$ as the output of self-consistent solutions of the coupled set of equations, which is reflected in the $|\vec{B}|$ profile.

In the present computations, we have assumed the flow to be vertically averaged without allowing any vertical component of the flow velocity (but keeping all the components of magnetic field). The most self-consistent approach, in order to understand vertical transport of matter through the magnetic effects which in turn leads to the radial infall of rest of the matter, is considering the flow to be moving in the vertical direction from the disk plane as well. Such an attempt, in the absence of magnetic effects, was made earlier by Bhattacharya et al. (2010) in the model framework of coupled disk-outflow systems. In such a framework, the authors furthermore showed that the outflow power of the correlated disk-outflow systems increases with the increasing spin of black holes. Our future goal is now to combine that model with the model of present work, so that the coupled disk-outflow systems can be investigated more self-consistently and rigorously, when the magnetic field plays indispensable role in order to generate vertical flux in the three-dimensional flows.

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